



Rethinking the Role of Gradient-Based Attribution Methods for Model Interpretability

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Implicit Density Models

The logits $f_i(x)$ of softmax-based models for class *i*...

 $p_{\theta}(y=i \mid x) = \frac{\exp f_i(x)}{\sum_i \exp f_j(x)} = \frac{p_{\theta}(x \mid y=i)p(y=i)}{\sum_i p_{\theta}(x \mid y=j)p(y=j)}$

... can be viewed as an energy function ...

$$p_{\theta}(x \mid y = i) = \frac{\exp f_i(x)}{Z}$$

...and the logit-gradients as gradients of the log density!

$$\nabla_{x} \log p_{\theta}(x \mid y = i) = \sum_{i=1}^{n} \nabla_{x} f_{i}(x)$$
gradients of log density logit-gradients

This leads to the following hypothesis:

Logit-gradients are highly structured because of their alignment with the ground truth gradients $\nabla_{x} \log p_{data}(x \mid y) \approx \nabla_{x} \log p_{\theta}(x \mid y) = \nabla_{x} f_{i}(x)$

Score-Matching

Score-Matching is a generative modelling principle based on aligning gradients of log density, by minimizing the following objective.

$$J(\theta) = \mathbb{E}_{p_{data}(\mathbf{x})} \frac{1}{2} \| \nabla_{\mathbf{x}} \log p_{\theta}(\mathbf{x}) - \nabla_{\mathbf{x}} \log p_{data}(\mathbf{x}) \|_{2}^{2}$$

This can be re-written as an objective which omits the unknown $\nabla_x \log p_{data}(x)$ term.

$$J(\theta) = \mathbb{E}_{p_{data}(\mathsf{x})} \left(\operatorname{trace}(\nabla_{\mathsf{x}}^{2} \log p_{\theta}(\mathbf{x})) + \frac{1}{2} \|\nabla_{\mathsf{x}} \log p_{\theta}(\mathbf{x})\|_{2}^{2} \right) + C$$

Interpretability \iff Generative Modelling Effect on Generative Modelling

Implicit density modelling perspective reveals generative modelling interpretations of the following methods ordinarily used for interpretability.

- Logit-gradients \iff gradients of log density
- Activation maximization of logits \iff MCMC sampling via Langevin dynamics
- Pixel perturbation test \iff density ratio test

Experimental Setup

Objective: Train models with different levels of gradient alignment by regularization, and study their effect on input-gradient interpretability.

$$\underbrace{\ell_{reg}(f(\mathbf{x}), i)}_{\text{regularized loss}} = \underbrace{\ell(f(\mathbf{x}), i)}_{\text{cross-entropy}} + \lambda \underbrace{R(\mathbf{x})}_{\text{Regularizer}}$$

Regularized Score-Matching: We propose relaxations of score-matching to overcome computational intractability of Hessian trace estimation, and stabilize the objective, which we use as a regularizer.

$$h(\mathbf{x}) := \frac{2}{\sigma^2} \mathbb{E}_{\boldsymbol{v} \sim \mathcal{N}(0,\sigma^2 \mathbf{I})} \left(f_i(\mathbf{x} + \boldsymbol{v}) - f_i(\mathbf{x}) \right)$$

$$R(\mathbf{x}) = \left(\underbrace{\overset{\text{Hessian-trace}}{h(\mathbf{x})}}_{\text{score-matching}} + \frac{1}{2} \underbrace{\overset{\text{gradient-norm}}{\|\nabla_{\mathbf{x}} f_i(\mathbf{x})\|_2^2}}_{\text{stability regularizer}} + \underbrace{\overset{10^{-4}}{\mu} h^2(\mathbf{x})}_{\text{stability regularizer}} \right)$$

Other Baselines: We use the following other baseline regularizations for comparison

- No regularization
- Anti-score-matching regularization, where hessian trace is maximized instead of being minimized
- Gradient norm regularization, where norm of input-gradients is minimized



We measure sample quality using the GAN-test scores (higher is better) on samples generated from the implicit density models via Langevin MCMC.

Model	GAN-test (%)
Baseline ResNet	59.47
+ Anti-Score-Matching	16.40
+ Gradient Norm-regularization	80.07
+ Score-Matching	72.75

Conclusion: Score-matching and gradient-norm regularization improves, while anti-score-matching deteriorates sample quality.

Effect on Interpretability

We measure a proxy for interpretability using the pixel perturbation test (higher is better) on the input-gradients of various models.



Conclusion: Score-matching and gradient-norm regularized models improves, while anti-score-matching deteriorates gradient interpretability.



